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TWO-DIMENSIONAL EFFECTS WITH THE FLOW OF A REACTIVE LIQUID WITH PROPERTIES VARYING WITH THE DEPTH OF THE CONVERSION

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During the course of chemical conversions, the mechanical properties of a reacting liquid can vary. Thus, polymerization processes are usually accompanied by a considerable increase in the viscosity. This leads to the appearance of specific hydrodynamic effects. Some of these are considered in the present article using the example of the simplest two-dimensional problem.

§1. The article considers the steady-state laminar flow of a reactive Newtonian liquid in a tube. The viscosity μ and the density ρ of the liquid, during the course of chemical conversions, vary from the values $\mu = \mu_0$ and $\rho = \rho_0$ to the values $\mu = \mu_1$ and $\rho = \rho_1$ for total conversion.

We shall assume that the temperature of the liquid is constant and that the effect of diffusion can be neglected. In this case, at a given point, the depth of the conversion and the mechanical properties of the liquid are determined only by the time t at which the liquid reaches the given point. The dependences $\mu = \mu(t)$ and $\rho = \rho(t)$ are the same as in the case

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of a motionless liquid and can therefore be regarded as known.

We assume that the change in the properties of the liquid with time can be approximated by the jumpwise dependence

$$\mu = \mu_0,\ \rho = \rho_0 \text{ for } t < t_0,$$
$$\mu = \mu_1,\ \rho = \rho_1 \text{ for } t \geqslant t_0. \tag{1.1}$$

This assumption conserves the principal features of the phenomenon under consideration and, at the same time, assures considerable mathematical simplifications. With this assumption, the spatial distribution of the mechanical properties of the liquid reduces to the existence of two regions, in each of which these properties are constant. The region in which the properties of the liquid do not differ from the starting values narrows with increasing distance from the start of the tube; the time in which the liquid reaches points lying on this boundary is equal to t_0. Within the framework of the present work, the time t_0 can be interpreted as an induction period, after which there is an instantaneous total chemical conversion. Therefore, in what follows, the elements of the liquid whose mechanical properties coincide with the properties of the liquid with total conversion will be arbitrarily called reacted (or reaction products), while elements of the liquid whose properties do not differ from the starting properties will be called unreacted, and the interface between them will be called the front of the reaction.

We introduce the following notation: V and W are, respectively, the axial and radial components of the velocity; U is the volumetric flow rate of the starting liquid, referred to unit area of the cross section; r is the relative distance from the axis of the tube; z is the distance from the start of the tube; P is the difference between the pressure at the inlet to the tube and the pressure at a given point; P_0 is the pressure drop between the inlet to and the outlet from the tube; z_0 and r_0 are the length and the radius of the tube, respectively.

Strictly speaking, the flow of the liquid should be considered not only in the tube itself, but before the inlet to and before the outlet from the tube. To avoid this, we shall assume that the distance l_e at which the perturbations arising at the inlet to the tube are damped is considerably less than the distance l_* at which the effect of a change in the properties of the liquid ($l_* \sim Ut_0$) becomes appreciable and that the pressure drop across the tube can be neglected.

In addition, we assume that the effect of inertial forces is small, that the velocity of the flow at any given point is directed practically along the axis of the tube ($W \ll V$), and that any appreciable changes in the picture of the flow take place at distances which so far exceed the width of the flow that in any given cross section changes in quantities taking place along the length of the tube can be neglected.

The greater the mass flow rate of the liquid, the greater the velocity of its motion ($V \sim U$). At the same time, the displacement of the liquid in a radial direction, arising as a result of a change in the mechanical properties of the liquid, is not connected directly with the value of the mass flow rate. Other conditions being equal, the value of the radial component of the velocity is determined by the width of the tube and by the rapidity of the change in the properties of the liquid ($W \sim r_0/t_0$). Consequently,

$$W/V \sim \varepsilon = r_0/Ut_0. \tag{1.2}$$

The effect of inertial forces in the given case is determined by the quantity

$$\rho W r_0/\mu \sim \mathrm{Re}_* = r_0(r_0/t_0)/(\mu_0/\rho_0), \tag{1.3}$$

which is an analog of the Reynolds number for radial displacement of the liquid, and, since for $l_e \ll l_*$, in accordance with [1],

$$l_e \sim r_0^2\rho_0 U/\mu_0, \text{ then } l_e/l_* \sim \mathrm{Re}_*.$$

Thus, the assumptions made mean that the discussion is limited to the principal term of the asymptotic curve as

$$\varepsilon \to 0 \text{ and } \mathrm{Re}_* \to 0. \tag{1.4}$$

The degree of smallness of the values of ε and Re_* required here in practice depends on the value of the relative change in the properties of the liquid and can be established from a comparison of the approximate and exact solutions.

In the approximation under consideration, the flow of the liquid in a tube of given length will not differ from the flow of a liquid in the corresponding section of a longer tube. A change in the mass flow rate of the liquid leads, essentially, only to a corresponding extension or compression of the picture of the flow. Therefore, in the solution of the problem, it is natural not to limit the discussion to any previously determined value of the length of the tube. Formally, this reduces to the use of dimensionless variables, not depending on z_0. Such variables are the quantities

$$v = \frac{V}{U}, \; w = W/\left(\frac{r_0}{t_0}\right), \; p = P/\left(8\frac{\mu_0}{r_0^2}U^2 t_0\right), \; \zeta = z/Ut_0, \tag{1.5}$$

which are, respectively, the axial and radial components of the velocity, the pressure, and the distance from the start of the tube. Here the introduction of a numerical factor into the definition of the dimensionless pressure brings the expression of the Poiseuille law to the simpler form $dp/d\zeta = 1$.

In these variables, the equations of motion of a viscous Newtonian liquid [2] assume the form

$$\gamma \mathrm{Re}_* \left\{v\frac{\partial v}{\partial \zeta} + w\frac{\partial v}{\partial r}\right\} = \frac{\partial}{\partial \zeta}\left\{8p + \varepsilon^2\left(\frac{2}{3}\nu - \nu'\right)\left(\frac{\partial v}{\partial \zeta} + \right.\right.$$

$$\left.\left. + \frac{1}{r}\frac{\partial}{\partial r} rw\right)\right\} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\nu\frac{\partial v}{\partial r}\right) + \varepsilon^2\left\{2\frac{\partial}{\partial \zeta}\left(\nu\frac{\partial v}{\partial \zeta}\right) + \frac{1}{r}\frac{\partial}{\partial r}\left(r\nu\frac{\partial w}{\partial r}\right)\right\}; \tag{1.6}$$

$$\varepsilon^2\gamma \mathrm{Re}_*\left\{v\frac{\partial w}{\partial \zeta} + w\frac{\partial w}{\partial r}\right\} = \frac{\partial}{\partial r}\left\{8p + \varepsilon^2\left(\frac{2}{3}\nu - \nu'\right)\left(\frac{\partial v}{\partial \zeta} + \right.\right.$$

$$\left.\left. + \frac{1}{r}\frac{\partial}{\partial r} rw\right)\right\} + \varepsilon^2\left\{\frac{2}{r}\frac{\partial}{\partial r}\left(r\nu\frac{\partial w}{\partial r}\right) - \frac{2\nu w}{r^2} + \frac{\partial}{\partial \zeta}\nu\left(\frac{\partial v}{\partial r} + \varepsilon^2\frac{\partial w}{\partial \zeta}\right)\right\}, \tag{1.7}$$

where $\gamma = \rho/\rho_0$; $\nu = \mu/\mu_0$; $\nu' = \mu'/\mu_0$; μ' is the coefficient of the volumetric viscosity; and the quantities ε and Re_* are defined in (1.2) and (1.3), respectively.

Passing to the limit in (1.4), from (1.7) we obtain $\partial p/\partial r = 0$, so that $p = p(\zeta)$, and from (1.6) it follows that

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\nu\frac{\partial v}{\partial r}\right) + 8dp/d\zeta = 0. \tag{1.8}$$

We assume that, in the cross section $\zeta = \text{const}$, the reacted liquid corresponds to $r \geq R$, i.e., in this cross section

$$\mu = \mu_0, \; \rho = \rho_0 \text{ for } r < R,$$
$$\mu = \mu_1, \; \rho = \rho_1 \text{ for } r \geqslant R, \; R = R(\zeta).$$

Then integrating (1.8) with respect to r and assuming that $v = 0$ for $r = 1$ and $\partial v/\partial r = 0$ for $r = 0$, for the distribution of the axial component of the velocity we will have

$$v\frac{1+\alpha}{2dp/d\zeta} = \begin{cases} 1 - r^2 + \alpha(R^2 - r^2) & \text{for } r < R, \\ 1 - r^2 & \text{for } r \geqslant R, \end{cases} \quad \alpha = \frac{\mu_1 - \mu_0}{\mu_0}, \tag{1.9}$$

The equation of continuity in the variables (1.5) has the form

$$\frac{1}{r}\frac{\partial}{\partial r}(r\gamma w) + \frac{\partial}{\partial \zeta}(\gamma v) = 0,$$

and the dimensionless stream function can be introduced:

$$\psi(\zeta, r) = \int_r^1 \gamma(\zeta, r')\, v(\zeta, r')\, 2r'dr', \tag{1.10}$$

in terms of which the components of the velocity are expressed using the relationships

$$w = \frac{1}{2\gamma r}\left(\frac{\partial\psi}{\partial\zeta}\right)_r,\quad v = -\frac{1}{2\gamma r}\left(\frac{\partial\psi}{\partial r}\right)_\zeta. \tag{1.11}$$

Substituting (1.9) into (1.10), for the dimensionless stream function ψ we obtain

$$\psi\frac{1+\alpha}{dp/d\zeta} = \begin{cases}(1-r^2)^2+\alpha(R^2-r^2)^2+\beta(1-R^2)^2 & \text{for } r<R,\\ (1+\beta)(1-r^2)^2 & \text{for } r\geqslant R,\end{cases} \tag{1.12}$$

$$\beta = \frac{\rho_1-\rho_0}{\rho_0}.$$

The mass flow rate of the liquid through any given cross section is exactly the same, and $\psi \equiv 1$ for $r = 0$. Therefore, for the gradient of the pressure, from (1.12) it follows that

$$dp/d\zeta = (1+\alpha)/[1+\alpha R^4+\beta(1-R^2)^2]. \tag{1.13}$$

The expressions given permit a complete determination of the characteristic curve of the flow in accordance with the dependence $R = R(\zeta)$. By the same token, solution of the problem in the approximation under consideration reduces to finding the position of the reaction front.

§2. In finding the position of the reaction front, it is convenient to introduce into the discussion the fraction $\sigma = \sigma(\zeta)$, which is the reacted liquid in the total mass flow of liquid through a given cross section,

$$\sigma(\zeta) = \int_{R^2(\zeta)}^{1}\gamma(\zeta, r)\,v(\zeta, r)\,dr^2 \equiv \psi(\zeta, R(\zeta)). \tag{2.1}$$

From (2.1), taking account of (1.12) and (1.13), we have

$$\sigma = (1+\beta)(1-R^2)^2/[1+\alpha R^4+\beta(1-R^2)^2], \tag{2.2}$$

and since there is a one-to-one connection between the values of σ and R, finding the dependence $\sigma = \sigma(\zeta)$ is equivalent to finding the dependence $R = R(\zeta)$.

In accordance with (1.1), the position of the reaction front is determined by the condition that the time in which the elements of the liquid reach the front is equal to t_0. Since the time dt in which an element of the liquid, moving along the flow line $\psi = \text{const}$, traverses the distance between cross sections corresponding to the values of σ and $\sigma + d\sigma$ satisfies the relationship

$$d\sigma = (d\sigma/d\zeta)\,v(\sigma, \psi)dt/t_0,$$

then for finding $d\sigma/d\zeta$ there follows the integral equation

$$\int_0^{\psi}\left(\frac{d\zeta}{d\sigma}\right)\frac{d\sigma}{v(\sigma,\psi)} = 1,\ 0\leqslant\psi\leqslant 1. \tag{2.3}$$

Here $v(\sigma, \psi)$ denotes the value of the axial component of the velocity at the point corresponding to the given values of σ and ψ. Substituting into (1.9) the distance from the axis of the tube to the given line of flow, found from (1.12), as a result, taking account of (1.13), (2.2), for this value of the axial component of the velocity we obtain

$$v\frac{1+\beta}{2}\frac{1-R^2}{\sqrt{\sigma}} = \begin{cases}\sqrt{\psi} & \text{for } \psi<\sigma,\\ \sqrt{(1+\delta)\psi-\delta\sigma} & \text{for } \psi\geqslant\sigma,\ \delta=\alpha+\alpha\beta+\beta.\end{cases} \tag{2.4}$$

Using (2.4), the integral equation (2.3) can be represented in the form

$$(1+\beta)\int_0^{\sqrt{\psi}}\left[(1-R^2)\frac{d\zeta}{d\sigma}\right]\frac{d\sqrt{\sigma}}{\sqrt{(1+\delta)\psi-\delta\sigma}} = 1, \tag{2.5}$$

from which it can be seen that, if the expression standing in square brackets in (2.5) is set equal to a constant, then, since

$$\int_0^{\sqrt{\psi}} \frac{d\sqrt{\sigma}}{\sqrt{(1+\delta)\psi-\delta\sigma}} = \int_0^1 \frac{dx}{\sqrt{(1+\delta)-\delta x^2}} = \frac{\operatorname{arctg}(\sqrt{\delta})}{\sqrt{\delta}},$$

the left-hand part of the integral equation will not depend on the value of ψ.

Consequently, the solution of the integral equation (2.3) is

$$\frac{d\sigma}{d\zeta} = (1+\beta)\frac{\operatorname{arctg}(\sqrt{\delta})}{\sqrt{\delta}}(1-R^2),$$

and, going back from the variable σ to the variable R, for finding the position of the reaction front we have

$$\frac{dR^2}{d\zeta} = -\frac{\operatorname{arctg}(\sqrt{\delta})}{2\sqrt{\delta}} \cdot \frac{[1+\alpha R^4+\beta(1-R^2)^2]^2}{1+\alpha R^2}. \tag{2.6}$$

Integrating (2.6), we find the coordinates of the reaction front and from (1.13), (2.6) we determine the change in the pressure along the length of the tube.

Setting $x = (1 - R^2)/(1 + \alpha R^2)$, the coordinates of the reaction front can be represented parametrically in the form

$$\zeta = \frac{\operatorname{arctg}(x\sqrt{\delta})}{\operatorname{arctg}(\sqrt{\delta})} + \frac{1+\alpha x}{1+\delta x^2}\frac{x\sqrt{\delta}}{\operatorname{arctg}(\sqrt{\delta})}, \quad R = \sqrt{\frac{1-x}{1+\alpha x}}, \tag{2.7}$$

and, with $\zeta \leq \zeta_*$, where

$$\zeta_* = 1 + \frac{1}{1+\beta}\frac{\sqrt{\delta}}{\operatorname{arctg}(\sqrt{\delta})} \tag{2.8}$$

is the maximal extension of the reaction front along the length of the tube, for the difference in the pressures at the inlet to the tube and at the given point we will have

$$p = \frac{5+2\alpha x+3\delta x^2}{4}\left(\frac{1+\alpha x}{1+\delta x^2}\right)^2 \frac{x\sqrt{\delta}}{\operatorname{arctg}(\sqrt{\delta})} + \frac{3}{4}\frac{\operatorname{arctg}(x\sqrt{\delta})}{\operatorname{arctg}(\sqrt{\delta})}\left\{1+\frac{\alpha^2}{\delta}\left[1-\frac{x\sqrt{\delta})}{\operatorname{arctg}(x\sqrt{\delta})}\right]\right\}. \tag{2.9}$$

The parameter x is the ratio of the value of the axial component of the velocity at the reaction front to the velocity of the flow of the liquid at the center of the tube

$$\frac{1}{2}v\frac{1+\delta x^2}{1+\alpha x} = \begin{cases} x+(1-x)\left(1-\dfrac{r^2}{R^2}\right) & \text{for } r<R, \\ x\dfrac{1-r^2}{1-R^2} & \text{for } r\geqslant R. \end{cases}$$

At the start of the tube, the reaction front is in contact with the walls, and the parameter x is equal to zero. With increasing distance from the start of the tube, the value of x rises and, with $\zeta = \zeta_*$, where the reaction front reaches the center of the tube, becomes equal to unity.

The fraction of the cross section occupied by reacted liquid $(1 - R^2)$ and the fractions σ and σ', which represent the reacted liquid, respectively, in the total mass flow rate and the volumetric mass flow rate of the liquid through the given cross section, are connected with the parameter x by the relationships

$$\sigma = \frac{1+\delta}{1+\delta x^2}x^2, \quad \sigma' = \frac{1+\alpha}{1+\alpha x^2}x^2, \quad 1-R^2 = \frac{1+\alpha}{1+\alpha x}x, \tag{2.10}$$

while, for the distribution of the radial component of the velocity, from (1.11) it follows that

$$rw\frac{2\sqrt{\delta}}{\operatorname{arctg}(\sqrt{\delta})}=\begin{cases}\dfrac{(1+\alpha)x^2}{(1+\alpha x)^2}(\alpha-\delta x)\left(\dfrac{1-r^2}{1-R^2}\right)^2 & \text{for } r>R,\\ \dfrac{r^2}{R^2}\dfrac{1-x}{(1+\alpha x)^2}\left[(1+\alpha)(\beta+\delta x)x-\right. & \\ \left.-(1-x)(\alpha-\delta x)\left(1-\dfrac{r^2}{R^2}\right)\right] & \text{for } r<R.\end{cases} \tag{2.11}$$

For $\zeta > \zeta_*$, there is the usual Poiseuille flow of the reacted liquid and

$$p = p_* + [(1+\alpha)/(1+\beta)](\zeta - \zeta_*),\ p_* = p(\zeta_*).$$

Depending on whether the density of the liquid decreases or increases during the course of the chemical conversions, the velocity of the flow for $\zeta \geq \zeta_*$ will be greater or less than at the start of the tube. However, the distance from the axis of the tube up to a given line of flow for $\zeta \geq \zeta_*$ is the same as at the start.

The behavior of the lines of flow for $\zeta < \zeta_*$, in accordance with (2.11), can be different. These differences are shown in Fig. 1, where the flow lines correspond to the solid lines, while the dashed lines show the position of the reaction front and the geometric location of the points at which the radial component of the velocity is equal to zero. The values of the quantities δ and β with which one case or another is realized are shown in Fig. 2, where the points lying below the straight line $\beta = \delta$ correspond to an increase in the viscosity with the depth of the conversion. Since the mass flow rate of the liquid is constant, an increase in the velocity leads to a situation in which the velocity of the flow of unreacted liquid increases and the flow lines extend to the axis of the tube. If the viscosity decreases, the velocities of the flow of unreacted liquid equalize out, and the flow lines diverge toward the walls.

The values of the parameter x distinguished in Fig. 1 are $x_\alpha = \alpha/\delta$, $x_\beta = -\beta/\delta$, and the value of x_0 satisfying the equation $\alpha\delta x^2 + 2\delta x - \alpha = 0$. Substituting these values into (2.7), it can be shown that, in cases a and d, $\zeta(x_0) = (1/2) \times \zeta(x_\alpha)$, and, in the cases b and e, $\zeta(x_0) = (1/2)\,[\zeta(x_\beta) + \zeta_*]$. In the cross section corresponding to $x = x_0$, the velocity of the flow at the center of the tube is equal to $v = \alpha/\sigma x_0$. For $\alpha > 0$, the value of the velocity is maximal, and for $\alpha < 0$, minimal.

If the properties of the liquid do not vary ($\alpha = \beta = 0$), then $w \equiv 0$, $\sigma = (1/4)\zeta^2$, $1 - R^2 = x = (1/2)\zeta$, $\zeta_* = 2$.

For $\mu_1 \ll \mu_0$, the reacted liquid plays the role of a lubricant. As a result, at the greater part of the reaction front, the unreacted liquid moves with a practically constant velocity $v \simeq 1$; the value of ζ_* tends toward unity.

When the viscosity of the reacted liquid significantly exceeds the initial ($\alpha \gg 1$), from (2.8) for the extended reaction front it follows that

$$\zeta_* \approx (2/\pi)\sqrt{I},\ I = (1+\alpha)/(1+\beta) = (\mu_1/\rho_1)/(\mu_0/\rho_0).$$

In this case, the flow of unreacted liquid is compressed into a narrow jet, breaking through the almost motionless layer of unreacted liquid. In the section of the formation of the jet, there is paractically no motion of the reaction products, and their presence at the wall leads to a situation in which the starting liquid moves as in a tube of variable cross section. In accordance with this, from (1.3) it follows that $dp/d\zeta \approx R^{-4}$, and passing to the limit $\alpha \to \infty$ in (2.7), (2.9), (2.10) with a fixed value of $\eta = \alpha x$, for this section we obtain

$$R^2 \approx 1/(1+\eta),\ \sigma \approx \eta^2/I,\ \zeta \approx (2/\pi\sqrt{I})\eta(\eta+2),\quad p \approx [\eta^2(\eta+2)^2 + 2\eta(\eta+2)]/\pi\sqrt{I}.$$

The corresponding limiting transitions show that for $I^{-1/2} \ll \zeta \leq \zeta_*$ the resistance to motion connected with the flow of the reacted liquid, which in each individual cross section is practically unaffected by the presence of a narrow jet of unreacted liquid at the center of the tube,

$$v \approx [2\sigma/(1+\beta)](1-r^2) \text{ for } r \geqslant R \simeq 0,\ dp/d\zeta \approx I\sigma.$$

The amount of reacted liquid forming in unit length of the tube does not depend on the mass flow rate and is constant along the length of the tube

$$t_0 d(\sigma U)/dz \equiv d\sigma/d\zeta \approx \pi/2\sqrt{I};$$

as a result, in the greater part of the extension of the front of the reaction

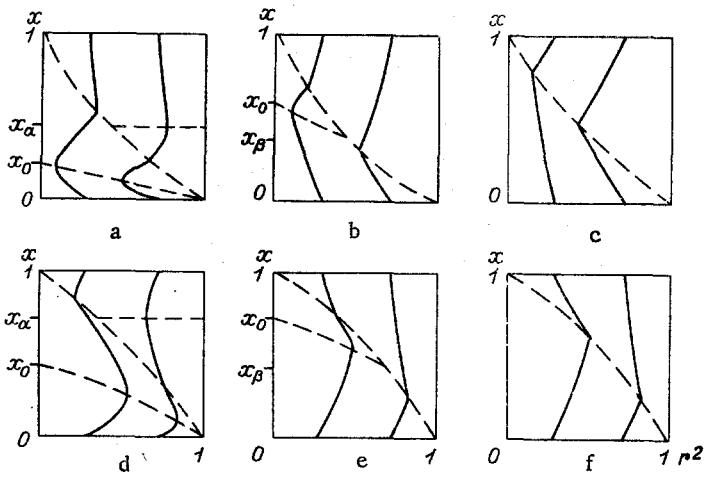

Fig. 1

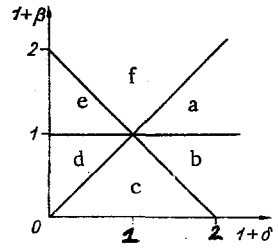

Fig. 2

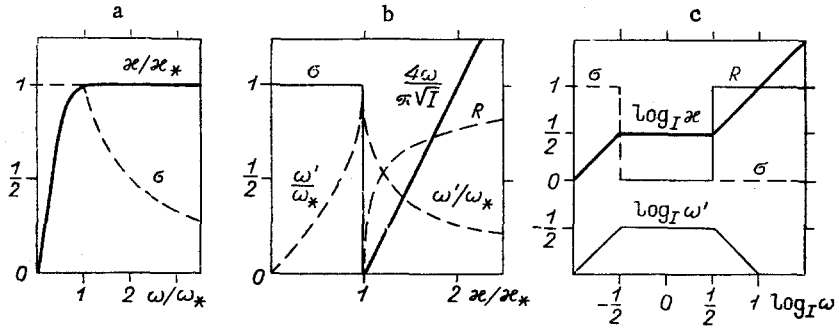

Fig. 3

$$\sigma \approx \zeta/\zeta_*,\ p \approx p_*(\zeta/\zeta_*)^2,\ p_* \approx (1/\pi)I^{3/2}.$$

To find the dependence between the limiting pressure and the mass flow rate of the liquid, we must go over from the variables (1.5) to the variables

$$\varkappa = p_0 \Big/ \left(8\,\frac{\mu_0}{r_0^2}\,\frac{z_0}{t_0}\,z_0\right),\ \omega = U \Big/ \left(\frac{z_0}{t_0}\right), \qquad (2.12)$$
$$\theta = \left(\frac{z_0}{U}\right) \Big/ t_0 \equiv \omega^{-1},$$

the scale values of which are connected, not with the value of the mass flow rate, but with the length of the tube. Here $\varkappa$ is the dimensionless pressure over the length of the tube; ω is the dimensionless mass flow rate of the liquid; and, if the density of the liquid does not vary, then $\theta = \omega^{-1}$ is the dimensionless mean residence time of the liquid in the tube.

A comparison of (2.12) and (1.5) shows that

$$\varkappa = \omega^2 p(\theta),\ p(\theta) \equiv p(\zeta) \text{ for } \zeta = \theta. \qquad (2.13)$$

The relationship (2.13) establishes a connection between the above-discussed change in the characteristics of the flow along the length of the tube and their dependence on the mass flow rate and the pressure drop.

The behavior of the system in the case where the viscosity of the reacted liquid is considerably greater than the starting viscosity (Fig. 3) is of great interest. The difference in the velocities of the motion of the reacted and unreacted liquid in this case is so great that, to achieve complete conversion, the mean residence time of the liquid in the tube must considerably exceed the induction period t_0. The maximal mass flow rate and pressure drop, with which there is still complete conversion at the outlet from the tube, are equal to

$$\omega_* \equiv \zeta_*^{-1} \approx \frac{\pi}{2\sqrt{I}},\ \varkappa_* \equiv \zeta_*^{-2} p_* \approx \frac{\pi}{4}\sqrt{I}.$$

With a mass flow rate greater than ω_*, the jet of unreacted liquid breaks through the whole tube, while the fraction of reacted liquid drops. However, so long as $\theta \gg I^{-1/2}$, i.e., right up to values of the residence time considerably less than the induction period, the amount of reacted liquid coming from the tube and the pressure drop practically do not vary:

$$\omega' \equiv \sigma\omega \simeq \omega_*,\ \varkappa/\varkappa_* \simeq 1.$$

If the motion of the liquid takes place with a given pressure drop, then with an increase in $\varkappa$ with a transition through $\varkappa/\varkappa_* = 1$, the mass flow rate of the liquid rises, and the mean residence time becomes somewhat less than the induction period; the motion of the reacted liquid practically ceases. The pressure drop for $\varkappa/\varkappa_* > 1$ is connected with the mass flow rate and the characteristics of the flow at the exit from the tube by the relationships

$$\varkappa \approx \varkappa_* + \omega,\ \frac{\varkappa_*}{\varkappa} \approx \frac{1-R^4}{1+R^4},\ \frac{\varkappa}{\varkappa_*} \approx \frac{1}{2}\left[\frac{\omega'}{\omega_*} + \frac{\omega_*}{\omega'}\right],\ \omega' < \omega_*,$$

while for $\varkappa/\varkappa_* < 1$

$$\frac{\varkappa}{\varkappa_*} \approx 2\frac{\omega}{\omega_*} - \frac{\omega^2}{\omega_*^2}.$$

The assumptions made with the solution of the hydrodynamic problem do not permit, in the present work, a discussion of the critical phenomena connected with the ambiguous character of the dependence of the mass flow rate of the liquid on the pressure drop [3].

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